Light Induced Hall effect in semiconductors with spin-orbit coupling

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We show that optically excited electrons by a circularly polarized light in a semiconductor with spin-orbit coupling subject to a weak electric field will carry a Hall current transverse to the electric field. This light induced Hall effect is a result of quantum interference of the light and the electric field, and can be viewed as a physical consequence of the spin current induced by the electric field. The light induced Hall conductance is calculated for the p-type GaAs bulk material, and the n-type and p-type quantum well structures.

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The phase coherent semiconductor spintronic device is an important candidate for the quantum devices, which allows the storage, manipulation and transport of quantum information[1]. Due to the quantum nature of a spin system, a single electron with spin 1/2 is an ideal qbit for quantum computing and an ideal unit for data storage. Therefore, the study of spin transport is very important for the future development of spintronic techniques. One of the most common methods in manipulating and detecting an electron's spin state is the optical absorption or emission of the circularly polarized light (CPL)[2]. The spin polarized charge current may be induced by the absorption of the CPL, which is called circular photogalvanic effect(CPGE)[3]. The CPGE was first proposed almost thirty years ago[4] and has been detected in both bulk materials and semiconductor quantum well(QW) structures[5].

In this Letter, we propose a new effect for a broad class of semiconductors with spin-orbit coupling, which we shall call light induced Hall effect (LIHE). In that effect, optically excited electrons by a CPL in the semiconductor subject to a weak static electric field will carry a Hall current transverse to the electric field. Different from the CPGE, which only occurs in materials with inversion asymmetry, the LIHE occurs also in bulk zincblende structure materials such as GaAs where the inversion symmetry is preserved[1]. The LIHE may be viewed as a response of the local spin Hall current induced by the electric field in the spin-orbit system to the CPL[6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In the materials with the structure inversion asymmetry the LIHE is expected to be more pronounced in the case where the incident light is normal to the sample to eliminate the CPGE[2]. In particular, we predict and estimate the effect by calculating the Hall photocurrent for three different systems: the p-type GaAs bulk material, the n-type and p-type GaAs quantum well structures.

We begin with a more detailed description of and general discussions on the LIHE, followed by explicit calculations of the effect on prototype systems. Let us consider a semiconductor with an incident CPL along the z-axis with $\vec{e_p}$ its Poynting unit vector, and a weak external static electric field $\vec{E}$ along the x-axis. Similar to the ordinary Hall effect, a transverse electric current along the y-direction will be generated in addition to the current along the x-direction. The schematic plot of the LIHE can be simply illustrated in fig1. The transverse current in this case is entirely induced by the CPL through the optical transition from the valence to the conduction band and its direction and magnitude can be determined by $\mathbf{J}_{hall} = \sigma_{xy}\lambda\mathbf{E}\times\mathbf{e}_P$, where σ_{xy} is the light induced Hall conductivity, $\lambda = \pm 1$ is the helicity of the CPL. From the symmetry point of view, the CPL in the LIHE plays the similar role as the magnetic field in the ordinary Hall effect to break the time reversal symmetry. However, unlike the ordinary Hall effect, the LIHE is purely a quantum effect induced by the spin-orbit coupling. As we will show below, the LIHE is induced by the Berry curvature of the band structure in the k-space.

LIHE may be understood as a quantum interference effect between the CPL and the static electric field. As discussed by Murakami et al.[6] and by Sinova et al.[7], when an electron (or a hole) moving along the y-direction is accelerated along the x-direction due to the electric field, its spin will tilt upward or downward along the z-direction. The electrons (or holes) moving with opposite momentum along the y-direction in the electric field will tilt their spins with one upward and the other downward, thus generating a non-zero spin current $j_y^z = 1/2(v_y\sigma^z + \sigma^z v_y)$. This spin Hall effect has generated a lot of research interest recently[6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In the presence of the right handed CPL, in the GaAs bulk or quantum well, the electrons will be pumped from the valence to the conduction band. Within the dipole approximation, only an electron with total angular momentum along the z-axis $J_z = -3/2$ ($J_z = -1/2$) will absorb the CPL and jump to the conduction band with $S_z = -1/2$ ($S_z = 1/2$). Therefore if the electron spin in the valence band tilt upwards or downwards, the corresponding transition rate to the conduction band will then be enhanced or suppressed due to the transition selection rule. As a consequence, the imbalance of the photo-excited electron density of the conduction band in the k space along the y-direction will also be induced, which leads to a spin polarized current along the y direction.

LIHE can also be viewed as the optical response of the

system carrying a pure spin current. The possible physical consequence induced by the spin current is a highly interesting issue in the field of spintronics. The LIHE generated by the spin current can then be either used to detect the existence of a spin current or to design the new type of quantum devices. The spin current generated by spin Hall effect is very difficult to detect. Up to now, the only way to measure the spin current flowing through the sample is to measure the spin accumulation at the edges generated by the spin current[16, 17]. Although the spin accumulation has been detected by two experiments using Kerr effect and photo luminescence spectra respectively, the quantitative relationship between the spin accumulation at the sample edge and the strength of the spin current flowing through the bulk is still not very clear. Since the measurement of the charge current can be relatively easily carried out by detecting the magnetic field built up around the current, for instance, the LIHE should shed light on the new methods of measuring spin current.

In what follows, we will discuss the LIHE in three different systems, namely the 3D hole system described by the Luttinger model, the 2D hole gas described by the Luttinger model under the confinement potential along the z-direction and the 2D electron gas described by the Rashba model.

We first consider a single particle Hamiltonian with momentum $\vec{P}$ in the bulk GaAs[18],

$$H_v(\overrightarrow{P}) = \frac{P^2}{2m}(\gamma_1 + \frac{5\gamma_2}{2}) - \frac{\gamma_2}{m}(\vec{S}\cdot\overrightarrow{P})^2 \quad (1)$$

$$H_c(\overrightarrow{P}) = \frac{P^2}{2m_e} \quad (2)$$

for a hole in the valence band and an electron in conduction band, respectively.

The above Hamiltonian can be easily diagonalized. To calculate the modification of the inter-band transition rate induced by the static electric field, we use a nonlinear response theory, where the second order correction combining the electric field $\overrightarrow{E}$ and the intensity of the light I will be taken into account. This high order response term can be obtained by the following way. First we switch off the light field and obtain the approximate wave function to the first order of the static electric field $\overrightarrow{E}$. Then we switch on the light field and use the $\overrightarrow{E}$ dependent wave function to calculate the transition rate. Following references[8, 21], the electric field is included in the Hamiltonian through the vector potential $\overrightarrow{A} = \overrightarrow{E}t$ and the momentum $\overrightarrow{P}$ in equation 2 and 1 is replaced by $\overrightarrow{P} - e\overrightarrow{E}t$.

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We assume the electric field is switched on at time $t = 0$ and obtain the first-order time-dependent wave function $|m,k,t\rangle^E$ for such a system in terms of the instantaneous eigenstates,

$$|m,\mathbf{k},t\rangle^E = \exp\left\{-i\int_0^t dt'\varepsilon_m(\mathbf{k},t')/\hbar\right\}\{|m,\mathbf{k},t\rangle + i\sum_{n\neq m} \frac{|n,\mathbf{k},t\rangle\,(f_{n,k}-f_{m,k})\,\mathbf{\Omega}_{nm}(\mathbf{k},t)\bullet e\mathbf{E}}{[\varepsilon_n(\mathbf{k},t)-\varepsilon_m(\mathbf{k},t)]}\left[1-e^{-i(\varepsilon_n(t)-\varepsilon_m(t))t/\hbar}\right]\} \quad (3)$$

where $|m,k,t\rangle$ is the instantaneous eigenstates of the Hamiltonian $H_v(\hbar\overrightarrow{k} - e\overrightarrow{E}t)$, which satisfies

$$H_v(\hbar\mathbf{k} - e\mathbf{E}t)\,|n,\mathbf{k},t\rangle = \varepsilon_n(t)\,|n,\mathbf{k},t\rangle \quad (4)$$

, $f_{n,k}$ is the Fermi distribution function and $\Omega_{nm}(\mathbf{k},t) = \langle n,\mathbf{k},t|\,\frac{\partial}{\partial\mathbf{k}}\,|m,\mathbf{k},t\rangle$ is the Berry curvature of the Bloch states.

We then switch on the CPL. The optical transition rate can be obtained by solving the time dependent Schrodinger equation perturbatively. After lengthy derivation, we can prove that the optical transition rate in the presence of the electric field can be obtained by simply using the above wavefunction in the Fermi golden rule, which reads

$$\Gamma_{mk,\alpha} \approx \{|\langle\alpha,\mathbf{k}|\,W\,|m,\mathbf{k}\rangle|^2 + 2\sum_{n\neq m} \mathbf{Re}[i\frac{W_{\alpha n}(\mathbf{k})\mathbf{\Omega}_{nm}(\mathbf{k},0)\,W_{m\alpha}(\mathbf{k})\bullet e\mathbf{E}\,(f_{n,k}-f_{m,k})}{[\varepsilon_n(\mathbf{k})-\varepsilon_m(\mathbf{k})]}]\} \times f_{m,k}\,(1-f_{\alpha,k})\,\delta\,(\hbar\omega-\varepsilon_\alpha(\mathbf{k})+\varepsilon_m(\mathbf{k})) \quad (5)$$

The matrix $\hat{W}$ describes the coupling between the electrons in the solid and the right handed CPL in the dipole approximation and takes the form of

$$\hat{W} = \begin{pmatrix} & 3/2 & -3/2 & 1/2 & -1/2 \\ 1/2 & g & 0 & 0 & 0 \\ -1/2 & 0 & 0 & \frac{g}{\sqrt{3}} & 0 \end{pmatrix}$$

where $g = dE_{rad}$ with d the effective dipole induced by the CPL and E_{rad} the amplitude of the electric field of the CPL. Assuming the power density of the CPL to be $100mW/mm^2$ and $d = 4.8\times10^{-29}C\cdot m$[22], we estimate the coupling energy of electron and CPL to be $2.6038\times10^{-6}eV$.

Within the simplest relaxation approximation, we can express the Hall photocurrent as the summation of the electron and hole currents,

$$\left\langle\overrightarrow{j}_{total}\right\rangle = \sum_{\mathbf{k}}\sum_{m\alpha}\left[e\overrightarrow{v}^{e}_{\alpha\alpha}(\mathbf{k})\,\Gamma_{mk,\alpha}\tau_e - e\overrightarrow{v}^{h}_{mm}(\mathbf{k})\,\Gamma_{mk,\alpha}\tau_h\right] \quad (6)$$

where $\overrightarrow{v}^{e}_{k}$ and $\overrightarrow{v}^{h}_{k}$ are the velocity operator, τ_e and τ_h are the relaxation time for the electrons and holes respectively. For circularly polarized light propagate perpendicular to the xy-plane, the contribution from the first

term in equation 5 cancels exactly after integrating over k. Therefore in the present case the total charge current,which is found along the y direction, is purely induced by the static external electric field. Similar to the Hall effect, we can express the transverse charge current in terms of the electric field, which reads $J_y = \sigma^{ph}_{xy} E_x$ with

$$\sigma^{ph}_{xy} = \sum_{m\alpha}\sum_{\mathbf{k}} \left[e v^{e}_{\alpha\alpha,x}(\mathbf{k})\,\Gamma_{mk,\alpha}\tau_e - e v^{h}_{mm,x}(\mathbf{k})\,\Gamma_{mk,\alpha}\tau_h \right] \times$$
$$\{2\sum_{n\neq m} \mathbf{Re}[i\frac{W_{\alpha n}(\mathbf{k})\mathbf{\Omega}_{nm}(\mathbf{k},0)\,W_{m\alpha}(\mathbf{k})\cdot\mathbf{e}_y\,(f_{n,k}-f_{m,k})}{[\varepsilon_n(\mathbf{k})-\varepsilon_m(\mathbf{k})]}$$
$$\times f_{m,k}\,(1-f_{\alpha,k})\,\delta\,(\hbar\omega-\varepsilon_\alpha(\mathbf{k})+\varepsilon_m(\mathbf{k}))\} \qquad (7)$$

For the 3D Luttinger model, we can solve the unperturbated Hamiltonian analytically and obtain a very simple analytical expression for the light induced Hall conductance in the low temperature as

$$\sigma^{ph}_{xy} = \sum_{m=L,H} \frac{3\pi^2+2}{16\hbar\omega m_c \pi} \frac{\alpha I e^2 \tau m_0 \hbar \left(f_{\tilde{m},k_m} - f_{m,k_m}\right)}{(\hbar\omega - E_g)\,\gamma_2 \mu_m}$$

where α is the optical absorption coefficient, which is around $10^4 cm^{-1}$for GaAs, I is the intensity of the light, $\hbar\omega$ is the energy of the photo, E_g is the energy gap of GaAs,k_m satisfies $\hbar\omega - \varepsilon_c(\mathbf{k}_m) + \varepsilon_m(\mathbf{k}_m) = 0$, τ is the momentum relaxation time for the electrons in the conduction band, m_0 is the bare electron mass, m_c is the effective mass for the conduction band and μ_m is the optical effective mass defined as $\mu_m^{-1} = m_c^{-1} + m_v^{-1}$. If we choose the typical experimental parameters for GaAs as $\alpha = 10^4 cm^{-1}, \tau_e = 10^{-12}s, I = 100mW/mm^2, E_g = 1.42eV, \hbar\omega = 1.67eV$, we can obtain the value of the light induced Hall conductance to be $7.5805\times10^{-3}\Omega^{-1}m^{-1}$.

We can also calculate the light induced Hall conductance defined in equation 7 for the quantum well structure as well. In this case, the applied CPL must be normal to the plane. Without the external electric field, the CPL can only induce a pure spin current within the plane[19, 20]. The only difference here is using the eigen states for the subbands in the above equation 7. In the present study, we calculate the light induced Hall conductance for both p-type and n-type quantum well samples numerically. The Hamiltonian of the GaAs quantum well structure can be written as

$$H_{v,well}(\overrightarrow{P}) = H_v(\overrightarrow{P}) + V(z) + \lambda_v\left(\overrightarrow{P}\times\overrightarrow{S}\right) \qquad (8)$$
$$H_{c,well}(\overrightarrow{P}) = H_c(\overrightarrow{P}) + V(z) + \lambda_c\left(\overrightarrow{P}\times\overrightarrow{\sigma}\right) \qquad (9)$$

where $H_v(\overrightarrow{P})$ and $H_c(\overrightarrow{P})$ are the Hamiltonian for the holes in the valence band and electrons in the conduction band, λ_v and λ_c are the effective Rashba coupling for the valence band and conduction band respectively which are induced by the structure inversion symmetry breaking, and $\overrightarrow{S}$, $\overrightarrow{\sigma}$ are the spin 3/2 and 1/2 matrices for valence band and conduction band respectively. In the present paper, we choose the confinement potential $V(z) = +\infty$ for $|z| > L$ and $V(z) = 0$ otherwise. Using the numerical techniques presented in detail in our previous paper[15], we first obtain the subband dispersions for the e1, HH1 and LH1 subbands, which are plotted in Fig1. In the calculation, we choose $\gamma_1 = 7.0$, $\gamma_2 = 1.9$, $m_e = 0.067m_0$, where m_0 is the bare electron mass. Then we calculate the light induced Hall conductance for both the p-type and n-type quantum well structure with the following parameters, $\tau_e = 5.8863\times10^{-11}s$, $I = 100mW/mm^2$, $g = 2.6038\times10^{-6}eV$. The carrier density is chosen to be $9.2807\times10^{10}cm^{-2}$ for the n-type case and 2. $6261\times10^{11}cm^{-2}$ for the p-type case. The results are shown in Fig2 and 3 respectively. In the present study, we only include the transition between the HH1,LH1 and e1 subbands. The difference behavior of the light induced Hall effect between the n-type and p-type samples quite clear in Fig2 and 3. In the n-type sample the contribution to the light induced Hall conductance from the $HH1-e1$ transition has a different sign with that of the $HH1-e1$ transition. While in the p-type sample, the contribution comes from two different transitions have the same sign. This interesting asymmetric behavior of n-type and p-type samples can be understood in the following way. In the n-type sample, the FS lies within the subband $e1$ and the spin of the electrons will tilt out of the plane when an electric field is applied. Suppose we use the right hand CPL here. According to the selection rule, for the HH1-e1 transition the only allowed process is from $|S_z = -3/2 >$ in the valence band to $|s_z = -1/2 >$ in the conduction band. And that of the LH1-e1 transition is from $|S_z = -1/2 >$ in the valence band to $|s_z = 1/2 >$ in the conduction band. Thus when the electron spin tilt out of the plane, the induced modification of the two transition rate will be opposite in sign , which gives the opposite sign for the light induced Hall

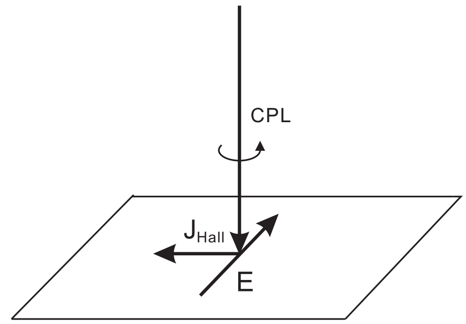

FIG. 1: The Hall photocurrent generated by the circularly polarized light and static electric field.

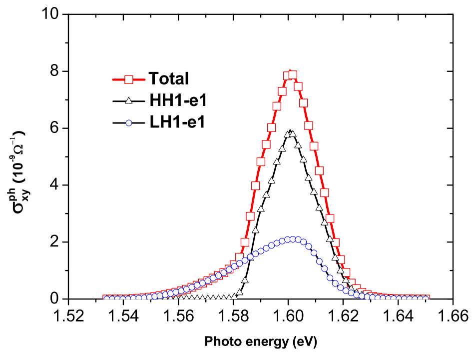

FIG. 2: The light induced Hall conductance in a p-type GaAs quantum well as the function of photon energy.

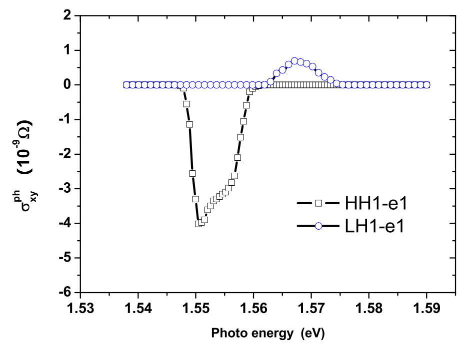

FIG. 3: The light induced Hall conductance in a n-type GaAs quantum well as the function of photon energy.

current.

From equation 7, we know that the strength of the LIHE is determined by the optical coupling matrix and the Berry curvature of the Bloch states at the manifold in the k-space which satisfy the energy conservation. The physical consequence of the Berry curvature in K-space was first found in the Anomalous Hall effect and late in the spin Hall effect. Then the LIHE we proposed here can be also viewed as a new physics consequence of the Berry curvature in K-space.

Another important issue we would like to discuss is the role of the disorder in the LIHE. As discussed in reference [12] and [14], if the spin orbital coupling in the system has the linear dependence in k, the disorder effect will exactly cancel the spin Hall effect in the thermal dynamic limit through the vertex correction terms. Therefore in such systems, the spin current induced by spin Hall effect only exists in the mesoscopic scale and vanishes in the macroscopic scale. Since the LIHE is generated by the direct optical absorption modulated by the static electric field, to see the LIHE it only requires the spin current to be generated in the scale of the light wave length, which is in the mesoscopic scale for the GaAs. Therefore, unlike the spin Hall effect, for such kind of systems, i.e. the n-type GaAs quantum well structure described by the Rashba model, the LIHE can also survive even for the macroscopic samples.

In summary, we have proposed a new effect, Light induced Hall effect in this paper. This effect is generated by the modulation of the optical transition rate in the k-space induced by the static electric field. The LIHE can be viewed in several different ways. First it can be viewed as the quantum interference effect between two different external fields, the light field and the static electric field. Secondly, the LIHE can be thought as the physics consequence generated by the non-zero spin current flowing through the bulk generated by the spin Hall effect.Thirdly, the LIHE can also be viewed as the physical effect reflecting the Berry curvature of the Bloch state in the k-space. We have also calculated the Light induced Hall conductance for three different semiconductor systems and made the quantitative predictions.

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